JHEP HYPER VERSION

RECEIVED: *June 6, 1998*, ACCEPTED: *June 21, 1998*

Absorption of fermions by D3-branes

Kazuo Hosomichi

Department of Physics, University of Tokyo,
Tokyo 113, Japan
E-mail: hosomiti@hep-th.phys.s.u-tokyo.ac.jp

ABSTRACT: The absorption cross section of dilatinos by D3-branes is calculated by means of both classical type IIB supergravity and the effective gauge field theory on their worldvolume. The two methods give the same results, supporting the microscopic description of black holes in terms of D-branes and giving another evidence of AdS/superconformal reciprocity.

KEYWORDS: D-branes, Black Holes in String Theory.

JHEP06(1998)009

Contents

1. Introduction

String theory or M theory are believed to give the microscopic description for black holes. One of the applications of this idea is to the Hawking process. Using the effective field theory on the intersection of D-branes or other solitons one can evaluate microscopically the absorption or emission of particles by black holes, which are semiclassically calculated by solving the field equations in black hole background. The success of this idea has lead to a new conjecture [1] that the large N limit of certain superconformal field theories are "dual" to the string theory or M theory on the product space of anti de-Sitter space and sphere.

In this paper we undertake the semiclassical and microscopic calculations of the absorption cross section of dilatinos by parallel D3-branes, where the absorption of scalars are calculated in some earlier works [2, 3, 4, 5]. We first calculate it from the solution of the Dirac equation in ten-dimensional type IIB supergravity on D3-brane background. Then we calculate it microscopically from the two-point function of certain operators in $D = 4, N = 4$ U(N) gauge theory on the worldvolume of D3-branes. The results of two calculations agree, giving further support to the idea of D-brane description of black holes and AdS/CFT correspondence.

2. Semiclassical calculation

In this section we calculate the absorption cross section by solving the Dirac equation of dilatinos in IIB supergravity. The extremal D3-brane solution in this theory is [6]:

$$ds^2_{(10)} = H^{-\frac{1}{2}}(-dx_0^2 + \cdots + dx_3^2) + H^{\frac{1}{2}}(dx_4^2 + \cdots + dx_9^2) \tag{2.1}$$

JHEP06(1998)009

$$C^{+}_{0123} = H^{-1} \tag{2.2}$$
$$H = 1 + \frac{4\pi g N}{r^4} = 1 + \frac{R^4}{r^4} \tag{2.3}$$

and we quote the Dirac equation from [7]:

$$\Gamma^{\mu} D_{\mu}\lambda = \frac{i}{960}\Gamma^{\rho_1\cdots\rho_5}\lambda F_{\rho_1\cdots\rho_5}\,. \tag{2.4}$$

Inserting the D3-brane solution into this equation gives

$$\left[H^{\frac{1}{2}}\Gamma^{a}\partial_a + \Gamma^{i}\partial_i + \frac{i}{4}\Gamma^{0123i}\partial_i(\ln H)\right](H^{\frac{1}{8}}\lambda) = 0\,, \tag{2.5}$$

where Γ^a, Γ^i are field-independent gamma-matrices and $a = \{0,1,2,3\}\,, i = \{4,\ldots,9\}$ are indices that run directions tangent or normal to the branes, respectively. We adopt $(-(+)^9)$ signature where Γ^0 is anti-hermite and $\Gamma^1,\cdots,\Gamma^9$ are hermite. The matrix Γ^{0123} has eigenvalues $\pm i$.

Next we shall find the spherical wave solution to (2.5). By the term 'spherical wave' we mean the wave that is spherical with respect to the six-dimensional spatial directions transverse to the branes. This can be done as in the case of lower space-time dimensions [8]. Firstly we introduce the orbital angular momentum and spin operators

$$L_{ij} = x_i\partial_j - x_j\partial_i\ ,\quad \Sigma_{ij} = \frac{1}{2}\Gamma_{ij}\,.$$

Then we put into (2.5) the spherical wave decomposition form for λ

$$H^{\frac{1}{8}}\lambda = e^{-i\omega t} r^{-\frac{5}{2}}\left\{F(r)\cdot\Psi_{-l} + iG(r)\cdot(\frac{\Gamma^0\Gamma^i x_i}{r})\Psi^{\pm}_{-l}\right\}\,, \tag{2.6}$$

where $\Psi^{\pm}_{-l}$ is the eigenspinor of total angular momentum with $(\Sigma_{ij}L_{ij}) = -l, \Gamma^{0123} = \pm i$. We can set the spatial momenta along the branes equal to zero by Lorentz transformations. We can easily obtain the radial wave equation

$$\begin{aligned}\omega H^{\frac{1}{2}}F + \left[\frac{d}{dr} + \frac{l+5/2}{r} \pm \frac{1}{4}(\ln H)'\right]G &= 0\,,\\ -\omega H^{\frac{1}{2}}G + \left[\frac{d}{dr} - \frac{l+5/2}{r} \mp \frac{1}{4}(\ln H)'\right]F &= 0\end{aligned} \tag{2.7}$$

and the expression for conserved flux

$$f \equiv i(F^{*}G - G^{*}F)\,. \tag{2.8}$$

The sign $\pm$ in (2.7) depends on whether we take Ψ^{+}_{-l} or Ψ^{-}_{-l} in (2.6).

We construct the approximate solution to (2.7) with the minus sign by the following procedure. We can rewrite (2.7) and (2.8) in terms of a new radial variable $x \equiv r/R$ and a new function ϕ as

$$\left[(x\frac{d}{dx})^2 - (l+2)^2 + \omega^2 R^2(x^2 + x^{-2})\right]\phi = 0 \tag{2.9}$$

$$f = i(x\frac{d}{dx}\phi^* \cdot \phi - \phi^* \cdot x\frac{d}{dx}\phi)\,. \tag{2.10}$$

In the asymptotic region or when $x \gg \omega R$ we can neglect the fourth term in the L.H.S of (2.9) and the approximate solution is expressed by the Bessel and Neumann functions as

$$\phi = aJ_{l+2}(\omega Rx) + bN_{l+2}(\omega Rx)\,. \tag{2.11}$$

In the horizon region or when $\omega Rx \ll 1$ the third term can be neglected in turn and the solution becomes

$$\phi = J_{l+2}(\omega Rx^{-1}) + iN_{l+2}(\omega Rx^{-1}) \tag{2.12}$$

up to multiplication by scalars. The solution is unique when the boundary condition is imposed so that ϕ has no outgoing flux on the horizon. By continuing these two solutions we can determine the coefficients a, b. The absorption cross section of plane wave can be calculated as follows:

$$\begin{aligned}\sigma_{\rm abs} &= \frac{4\pi^2}{3\omega^5}\frac{(l+4)!}{l!}\cdot\frac{f_{\rm abs}}{f_{\rm in}} \\ &= \frac{(l+4)!}{l![(l+1)!(l+2)!]^2}\frac{16\pi^4}{3\omega^5}\left(\frac{\omega R}{2}\right)^{4l+8}\,.\end{aligned} \tag{2.13}$$

On the contrary, solving the equation (2.7) with the plus sign gives

$$\sigma_{\rm abs} = \frac{(l+4)!}{l![(l+2)!(l+3)!]^2}\frac{16\pi^4}{3\omega^5}\left(\frac{\omega R}{2}\right)^{4l+12}\,, \tag{2.14}$$

which says that the absorption of these modes are suppressed by a factor of ω^4. In the s-wave case (2.13) becomes as

$$\sigma_{\rm abs} = \frac{\pi^4\omega^3R^8}{8} = \frac{G_{10}N^2\omega^3}{4}\,. \tag{2.15}$$

3. Microscopic calculation

In this section we shall calculate the absorption cross section microscopically. To begin with, we quote the action describing the motion of a D3-brane in generic type IIB supergravity background [9, 10, 11]

$$\begin{aligned} I &= I_{\rm DBI} + I_{\rm WZ} \\ &= -\int d^4x\sqrt{-\det(g_{ij} + e^{-\frac{\phi}{2}}\mathcal{F}_{ij})} + \int e^{\mathcal{F}}\wedge\mathcal{C}\,,\end{aligned} \tag{3.1}$$

where

$$g_{ij} = E_i^aE_j^b\eta_{ab}\,, \qquad \mathcal{F}_{ij} = F_{ij} - B_{ij}^{\rm (NS)}$$

and $\mathcal{C}$ is the collection of RR forms. The pull-back of superforms is defined, for example, as

$$B_{ij}^{\rm (NS)} \equiv E_j^BE_i^AB_{AB}^{\rm (NS)}\,. \tag{3.2}$$

Putting the on-shell superfield configuration into (3.1) and expanding it with respect to the fermionic coordinate $\theta^\alpha, \bar{\theta}^{\bar{\alpha}}$, it becomes, in the static gauge, as

$$\begin{aligned}\mathcal{L} = -1 &+ \frac{1}{2}\delta_{IJ}\partial X^I \partial X^J + \frac{i}{2}(\theta\sigma^i\partial_i\bar{\theta} + \bar{\theta}\sigma^i\partial_i\theta) - \frac{1}{4}e^{-\phi}F_{ij}F^{ij} + \frac{1}{4}\chi F_{ij}\tilde{F}^{ij}\\ &+(\text{nonrenormalizable terms})\\ &+(\text{terms containing the external fields})\,, \end{aligned} \tag{3.3}$$

where $i, j = 0, 1, 2, 3$ and $I, J = 4, \ldots, 9$ are indices of tangential and normal directions to the brane and σ^m $(m = 0, 1, \ldots, 9)$ are 16×16 matrices from which the ten-dimensional Dirac matrices are constructed.[1] By the term 'external fields' we mean the fields of type IIB supergravity in the bulk.

The above action contains twice the fermionic degree of freedom as is needed. We can reduce them by using κ-symmetry. The κ-transformation acts on the fermionic coordinates as

$$\delta_\kappa Z^M E_M^\alpha = \kappa^\alpha\ , \quad \delta_\kappa Z^M E_M^{\bar{\alpha}} = \bar{\kappa}^{\bar{\alpha}}$$

and κ satisfies, to the zeroth order in the world-volume fields, the following conditions:

$$\Gamma\kappa = -i\kappa\,, \qquad \Gamma\bar{\kappa} = i\bar{\kappa}\,,$$

$$\Gamma = \frac{1}{4!\sqrt{-g}}\varepsilon^{ijkl}\hat{\sigma}_{ijkl} = \hat{\sigma}_{0123}\,.$$

We shall hereafter call spinors with $\sigma_{0123} = \pm i$ or $\hat{\sigma}_{0123} = \pm i$ as the Left-(Right-)handed spinors in the four-dimensional sense. We take the following κ-gauge choice:

$$\theta^\alpha = \theta^\alpha_{\mathrm{L}}\,, \qquad \bar{\theta}^{\bar{\alpha}} = \bar{\theta}^{\bar{\alpha}}_{\mathrm{R}}\,. \tag{3.4}$$

The bulk dilatinos couple to the fields on the worldvolume as

$$\begin{aligned}\mathcal{L}_{\mathrm{int}} &= e^{-\frac{\phi_0}{2}}\theta_{\mathrm{L}}\sigma_{ij}F^{ij}\bar{\lambda} - e^{-\frac{\phi_0}{2}}\bar{\theta}_{\mathrm{R}}\sigma_{ij}F^{ij}\lambda\\ &= \mathcal{O}^{\bar{\alpha}}_{[\bar{\lambda}]}\bar{\lambda}_{\bar{\alpha}} + \mathcal{O}^{\alpha}_{[\lambda]}\lambda_\alpha\,,\end{aligned} \tag{3.5}$$

where ϕ_0 is the VEV of dilaton. Here we have neglected the terms of higher order in the external fields and the terms containing nonrenormalizable operators of the worldvolume gauge theory. One can find these interactions by considering the following terms in (3.1):

$$\frac{1}{2}e^{-\phi}F^{ij}B^{\mathrm{NS}}_{ij} + \frac{1}{2}\tilde{F}^{ij}B^{\mathrm{R}}_{ij}$$

and extracting terms linear in $\theta, \bar{\theta}$. There is no other contributions to this order. One can easily find out from (3.5) that only the following components:

$$\lambda_{\mathrm{R}}\ , \quad \bar{\lambda}_{\mathrm{L}}$$

[1] We follow largely the convention of [12]. Definitions and formulas which are relevant in this article are summarized in the appendix.

JHEP06(1998)009

couple to operators of dimension 7/2 on the worldvolume. This is consistent with the semiclassical result that the absorption of dilatinos are enhanced or suppressed according to the signs of their interaction with five-form field strength.

As is discussed in [13, 14, 4], one can calculate the absorption cross section from the 'discontinuity' of two point function of the operator that couples to the relevant external field. Let us calculate it for the case of the absorption of dilatinos. Using the following propagators:

$$
\begin{aligned}
\left\langle X^I(x)X^J(0)\right\rangle &= -i\delta^{IJ}G_F(x)\,,\\
e^{-\phi_0}\left\langle A_i(x)A_j(0)\right\rangle &= i\eta_{ij}G_F(x)\,,\\
\left\langle \theta^\alpha_{\rm L}(x)\bar\theta^{\bar\alpha}_{\rm R}(0)\right\rangle &= (P_{\rm L}\hat\sigma^m P_{\rm R})^{\alpha\bar\alpha}\partial_m G_F(x)\,,\\
G_F(x) &= \frac{1}{4i\pi^2}\frac{1}{x^2}\,,
\end{aligned}
\tag{3.6}
$$

we can easily calculate the two-point function

$$
\begin{aligned}
\Pi^{\alpha\bar\alpha}_{[\lambda]}(x) &\equiv \left\langle \mathcal{O}^\alpha_{[\lambda]}(x)\mathcal{O}^{\bar\alpha}_{[\bar\lambda]}(0)\right\rangle\\
&= -4i(\hat\sigma^m)^{\alpha\bar\alpha}\partial_m\partial^2 G_F(x)^2\,.
\end{aligned}
\tag{3.7}
$$

The Fourier transform of $\Pi^{\alpha\bar\alpha}_{[\lambda]}(x)$ is defined as

$$
\Pi^{\alpha\bar\alpha}_{[\lambda]}(p) = \int d^4x\,\Pi^{\alpha\bar\alpha}_{[\lambda]}(x)e^{ipx}
$$

and its 'discontinuity across the real axis' is calculated as

$$
\begin{aligned}
\Sigma^{\alpha\bar\alpha}_{[\lambda]} &\equiv \frac{1}{2i\omega}\left[\Pi^{\alpha\bar\alpha}_{[\lambda]}(p)|_{p^0=\omega+i\epsilon,p^i=0} - \Pi^{\alpha\bar\alpha}_{[\lambda]}(p)|_{p^0=\omega-i\epsilon,p^i=0}\right]\\
&= \frac{N^2}{4\pi}(\hat{\sigma^0})^{\alpha\bar\alpha}\omega^2\,.
\end{aligned}
\tag{3.8}
$$

In the above expression we have incorporated a factor N^2 that arises from multiple D3-branes. In fact we do not know the precise form of the action for multiple D3-branes, but the factor N^2 arises naturally when we assume that the effective theory on the worldvolume of N coincident D3-branes be $U(N)$ gauge theory and adopt the symmetrized trace prescription of [15].

As for the bulk dilatino states, we write down the second quantized form for $\lambda_\alpha, \bar\lambda_{\bar\alpha}$:

$$
\begin{aligned}
\lambda_\alpha(x) &= \sqrt{\frac{16\pi G_{10}}{32}}\int\frac{d^6\mathbf{k}}{(2\pi)^6 2\omega}\sum_i[b(\mathbf{k},i)u_\alpha(\mathbf{k},i)e^{-ikx} + d^\dagger(\mathbf{k},i)v_\alpha(\mathbf{k},i)e^{ikx}]\,,\\
\bar\lambda_{\bar\alpha}(x) &= \sqrt{\frac{16\pi G_{10}}{32}}\int\frac{d^6\mathbf{k}}{(2\pi)^6 2\omega}\sum_i[b^\dagger(\mathbf{k},i)u^*_{\bar\alpha}(\mathbf{k},i)e^{ikx} + d(\mathbf{k},i)v^*_{\bar\alpha}(\mathbf{k},i)e^{-ikx}]\,,
\end{aligned}
\tag{3.9}
$$

where x and k are seven-dimensional ones. The factor $(16\pi G_{10}/32)$ relates the dilatinos in [12] to the canonically-normalized ones. The canonical anticommutation relation reads

$$
\left\{\lambda_\alpha(x), \bar\lambda_{\bar\alpha}(y)\right\}|_{x^0=y^0} = (\frac{16\pi G_{10}}{32})\delta_{\alpha\bar\alpha}\delta^6(\mathbf{x}-\mathbf{y})
$$

$$\left\{b(\mathbf{k},i),b^\dagger(\mathbf{k}',j)\right\} = \left\{d(\mathbf{k},i),d^\dagger(\mathbf{k}',j)\right\} = (2\pi)^6 2\omega\delta^6(\mathbf{k}-\mathbf{k}')\delta_{ij} \tag{3.10}$$

and the spinor wave functions $u_\alpha(\mathbf{k},i), v_\alpha(\mathbf{k},i)$ satisfy

$$\begin{aligned}\sum_\alpha u^*_\alpha(\mathbf{k},i)u_\alpha(\mathbf{k},j) &= \sum_\alpha v^*_\alpha(\mathbf{k},i)v_\alpha(\mathbf{k},j) = 2\omega\delta_{ij}\,,\\ \sum_\alpha u^*_\alpha(\mathbf{k},i)v_\alpha(-\mathbf{k},j) &= \sum_\alpha v^*_\alpha(\mathbf{k},i)u_\alpha(-\mathbf{k},j) = 0\,,\\ \sum_i u_\alpha(\mathbf{k},i)u^*_{\bar\alpha}(\mathbf{k},i) &= \sum_i v_\alpha(\mathbf{k},i)v^*_{\bar\alpha}(\mathbf{k},i) = k_m(\sigma^m)_{\alpha\bar\alpha}\,.\end{aligned} \tag{3.11}$$

The absorption cross section of dilatinos σ_{abs} is calculated as

$$\begin{aligned}\sigma_{\mathrm{abs}}\delta_{ij} &= \left(\frac{16\pi G_{10}}{32}\right)\Sigma^{\alpha\bar\alpha}_{[\lambda]}u^*_{\bar\alpha}(\mathbf{k},i)u_\alpha(\mathbf{k},j)\\ &= \delta_{ij}\frac{G_{10}N^2\omega^3}{4}\,.\end{aligned} \tag{3.12}$$

This agrees with the macroscopic result (2.15).

4. Discussions

We have calculated in this paper the absorption cross section of dilatinos in two different ways. The two results agree, to the leading order in ω, both presenting the remarkable feature that the absorption is enhanced or suppressed according to their signs of interaction with five-form field strength.

Notice that in doing perturbative analysis of gauge theory we assumed $gN \ll 1$ and dropped terms of higher order in gN in calculating the two-point function. But this is exactly when R or the size of the black 3-brane is much smaller than the string scale and semiclassical approximation is not good. To see the correspondence of gauge theory and supergravity one should properly treat the non-abelian nature of the worldvolume theory and analyze the theory with large gN. Our result suggests that the value of the two point function $\left\langle \mathcal{O}^\alpha_{[\lambda]}(x)\mathcal{O}^{\bar\alpha}_{[\bar\lambda]}(0)\right\rangle$ remain the same at large gN. This does not seem obvious to us.

Checking the coincidence for the gravitinos and particles with other spin would be interesting. It seems that microscopic evaluation of the absorption of gravitinos is much more involved than in the case of dilatinos. One reason for this is the κ-symmetry. Expanding the D3-brane action (3.1) with respect to θ, one can find that the interaction terms by which the gravitinos couple to the worldvolume operators of the lowest dimension are

$$(i\theta\sigma_i\bar\psi^i + i\bar\theta\sigma_i\psi^i) + \frac{1}{6}\epsilon^{ijkl}(\bar\theta\sigma_{ijk}\psi_l - \theta\sigma_{ijk}\bar\psi_l)\,.$$

These terms come from I_{DBI} and $\int C_{0123}$, respectively. With our gauge-fixing these two terms cancel, but for other gauge choices such as proposed in [16] these terms don't vanish in general. It is somewhat strange, since the coupling of bulk fields and worldvolume fields seems κ-gauge-dependent, although the above interaction may yield no discontinuity to the two-point function and is invisible by the analysis of absorption cross section. Anyway we believe that a careful analysis will lead to a κ-invariant expression for absorption cross section of gravitinos as in the case of dilatinos.

A recent work [17] revealed that the field equation of a minimal scalar in D3-brane background can be exactly solved in terms of Mathieu functions. It seems that by applying this observation to the fermionic case one can get further implications on the world-volume theory of D3-branes.

Some other recent works [18, 19] claim that one can obtain all the Kaluza-Klein modes of AdS supergravity by expanding D3-brane action around AdS background. It would be interesting in this regard to see how higher-dimensional fermionic operators work out.

Acknowledgments

The author thanks T. Eguchi for introducing to this subject, Y. Sugawara, T. Kawano and K. Okuyama for encouragements and discussions. The author is also grateful for S. Das for communication and discussion on recent issues. The author is supported in part by JSPS Research Fellowships for Young Scientists.

A. Spinors in ten dimensions

We follow and use the convention of [12]. We use the representation in which the Gamma matrix can be expressed as

$$\Gamma^a = \begin{pmatrix} 0 & (\sigma^a)_{\alpha\beta}\,, \\ (\hat{\sigma}^a)^{\alpha\beta} & 0 \end{pmatrix}\,,$$
$$\Gamma^{11} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,, \tag{A.1}$$

where the 16×16 matrices $\sigma^a, \hat{\sigma}^a$ are real symmetric and satisfy the following identity:

$$(\sigma^a) = (1, \sigma^i)\,, \quad (\hat{\sigma}^a) = (1, -\sigma^i)\,, \qquad \left\{\sigma^i, \sigma^j\right\} = 2\delta^{ij}\,, \qquad i = 1, \ldots, 9.$$

Antisymmetric products of σ-matrices are defined as

$$\sigma^{ab\cdots c} = \sigma^{[a}\hat{\sigma}^b \cdots {}^{c]}\,, \qquad \hat{\sigma}^{ab\cdots c} = \hat{\sigma}^{[a}\sigma^b \cdots {}^{c]}\,.$$

Ten-dimensional Dirac spinor has 32 components and according to the eigenvalue of Γ^{11} it is decomposed into left-handed and right-handed spinors:

$$\psi = \psi_{\rm L} + \psi_{\rm R}\,,$$
$$\psi_{\rm L} = \frac{1}{2}(1 + \Gamma^{11})\psi = \begin{pmatrix} \varphi_\alpha \\ 0 \end{pmatrix}\,,$$
$$\psi_{\rm R} = \frac{1}{2}(1 - \Gamma^{11})\psi = \begin{pmatrix} 0 \\ \chi^\alpha \end{pmatrix}\,. \tag{A.2}$$

References

[1] J. Maldacena, *The Large N Limit of Superconformal Field Theories and Supergravity*, hep-th/9711200.

[2] I. Klebanov, *Nucl. Phys.* **B 496** (1997) 231, hep-th/9702076.

[3] S. Gubser, I. Klebanov and A. Tseytlin, *Nucl. Phys.* **B 499** (1997) 217 [hep-th/9703040].

[4] S. Gubser and I. Klebanov, *Phys. Lett.* **B 413** (1997) 41 [hep-th/9708005].

[5] S. Gubser, A. Hashimoto, I. Klebanov and M. Krasnitz, hep-th/9803023.

[6] G. Horowitz and A. Strominger, *Nucl. Phys.* **B 360** (1991) 197.

[7] J. Schwarz, *Nucl. Phys.* **B 226** (1983) 269.

[8] K. Hosomichi, hep-th/9711072.

[9] M. Cederwall, A. von Gussich, B. Nilsson and A. Westerberg, *Nucl. Phys.* **B 490** (1997) 163 [hep-th/9610148].

[10] M. Cederwall, A. von Gussich, B. Nilsson, P. Sundell and A. Westerberg, *Nucl. Phys.* **B 490** (1997) 179 [hep-th/9611159].

[11] E. Bergshoeff and P. K. Townsend, *Nucl. Phys.* **B 490** (1997) 145 [hep-th/9611173].

[12] P. Howe and P. West, *Nucl. Phys.* **B 238** (1984) 181.

[13] J. Maldacena and A. Strominger, *Phys. Rev.* **D 56** (1997) 4975 [hep-th/9702015].

[14] S. Gubser, *Phys. Rev.* **D 56** (1997) 7854 [hep-th/9706100].

[15] A. Tseytlin, *Nucl. Phys.* **B 501** (1997) 41 [hep-th/9701125].

[16] E. Bergshoeff, R. Kallosh, T. Ortin and G. Papadopoulos, *Nucl. Phys.* **B 502** (1997) 149 [hep-th/9705040].

[17] S. Gubser and A. Hashimoto, hep-th/9805140.

[18] S. Das and S. Trivedi, hep-th/9804149.

[19] S. Ferrara, M. Lledo and A. Zaffaroni, hep-th/9805082.